Robust Smith-Predictor Controller for Uncertain Delay Systems

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Necessary and sufficient conditions for the robust stability and performance of the Smith-predictor controller, modeled under a norm-bounded uncertainty, are given in a general formulation. In addition, a practical stability condition is obtained as a special corollary of the main results. These conditions provide useful and practical guidelines for the development of a systematic robust design method. In particular, an application is developed for the robust control of first-order deadtime systems with simultaneous uncertainties in all three parameters of the model. A simulation example and the results of a case study on the robust level control of a coupled-tanks apparatus are provided for illustration.

Introduction

Time delays are common phenomena in many industrial processes, and they cause complications in the control-related problems associated with these processes. Conventional controllers, like the proportional integral derivative (PID), are often ineffective for the control of such processes, as a significant amount of detuning would be required to maintain closed-loop stability. Smith (1957) introduced a deadtime compensator as shown in Figure 1, more commonly known now as the Smith-predictor controller. The controller incorporates a model of the process, thus allowing for a prediction of the process variables, and the controller may then be designed as though the process is delay free. The Smith-predictor controller offers potential improvement in the closed-loop performance over conventional controllers (Marshall, 1979), and it has been extended to multivariable systems with deadtimes (Alevisakis and Seborg, 1974; Palmor and Halevi, 1983). Palmor and Powers (1985) and Huang et al. (1990) have modified the Smith system to attain good load disturbance rejection properties.

However, like other model-based control systems, the Smith-predictor controller requires a good model of the process. In the face of inevitable mismatch between the model and the actual process, the closed-loop performance may be very poor. In fact, it has been shown that Smith systems may become unstable even for infinitesimal perturbation in the process dynamics (Palmor, 1980). This sensitivity problem has greatly deterred their otherwise potentially widespread applications in the industry, and it has resulted in detuned potential integral (PI) control continuing to be used for deadtime dominant processes.

Some work has been done in relation to the robustness issues of the Smith system. In Ioannides et al. (1979), stability boundaries were plotted as functions of error in a single plant parameter. Santacesaria and Scattolini (1993) developed a

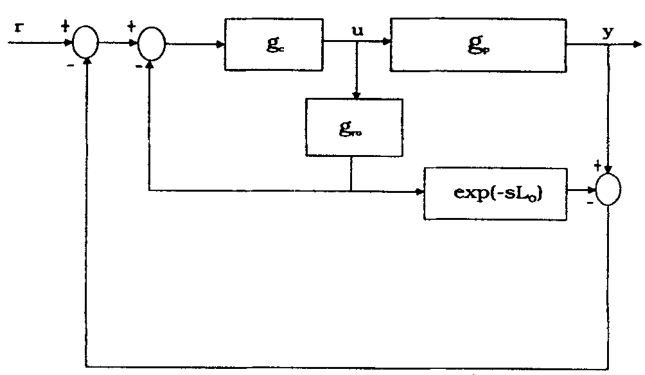

Figure 1. Smith-predictor controller.

"universal abacus" for selection of the closed-loop bandwidth, given an uncertainty bound on the process deadtime. However, all these studies have failed to consider the effect of simultaneous uncertainties in gain, time-constant, and deadtime on the robust performance of Smith-predictor controllers. Laughlin et al. (1987) addressed this problem by mapping the parametric uncertainties of a first-order model to convex hulls on the complex plane, and iteratively detuning the controller while visually inspecting the transformed uncertainty regions for robust performance. These procedures are computationally intensive and rather cumbersome to implement in a practical way. Yamanaka and Shimemura (1987) have derived conditions for the stability of the Smith systems under infinitesimal perturbations, but the stability problems under general uncertainty conditions remain unaddressed.

In this article, we investigate the robust stability and the robust performance of the Smith-predictor controller by using an equivalent internal model control (IMC) representation of the controller, and by drawing on existing results for IMC. Necessary and sufficient conditions for the robustness of the Smith system are developed in a general formulation under mild assumptions, and a practical stability condition is obtained as a special corollary. These conditions provide useful and practical guidelines for the design of a robust Smith system using a systematic two-step approach. First, an appropriate model and controller structure are selected to satisfy the practical stability condition and the parameters of the model computed from some process identification experiments (Ljung, 1987; Luyben, 1991). Next, the optimal parameters of the controller are chosen such that the ideal closed-loop transfer function will satisfy the performance specifications under perturbed conditions. A robust Smith system design is then developed for the first-order deadtime system with simultaneous uncertainties in all three parameters of the model. A general example and a case study on the robust level control of a coupled-tanks apparatus are provided to illustrate the design principles.

The article is organized as follows. The following section reviews the Smith-predictor controller. The robust stability, the robust performance, and the practical stability of the Smith system are investigated in the third section. Application of the design principles to the first-order deadtime systems is described in the fourth section, and a simulation example is provided for illustration. Finally, the results of the case study on the robust level control problem of a coupled-tanks apparatus are described in the fifth section.

Smith-Predictor Controller: a Review

The Smith controller was proposed by Smith (1957). Assume that a model for the process $g_p(s)$ is available; it is described by

$$g_{po}(s) = g_{ro}(s)e^{-sL_o},$$

where $g_{ro}(s)$ is a delay-free rational function.

The structure of the Smith-predictor controller is shown in Figure 1. It can be shown that the closed-loop transfer function is given by

$$g_{yr}(s) = \frac{g_c(s)g_p(s)}{1+g_c(s)\left[g_{ro}(s)-g_{ro}(s)e^{-sL_o}+g_p(s)\right]}. \quad (1)$$

In the case of perfect modeling, that is, $g_{po}(s) = g_p(s)$, the closed-loop transfer function between the setpoint and output is given by

$$g_{yr}(s) = g_{yr}^*(s) = \frac{g_c(s)g_{ro}(s)}{1+g_c(s)g_{ro}(s)}e^{-L_o s}. \quad (2)$$

This implies that the characteristic equation is free of the delay so that the primary controller $g_c(s)$ can be designed with respect to $g_{ro}(s)$. The achievable performance can thus be greatly improved over a conventional system without the delay-free output prediction.

Robustness Analysis

In the real world where the model does not represent the process exactly, *nominal stability* and *nominal performance* are not sufficient. It is shown by an example in Palmor (1980) that the Smith-predictor controller could be unstable for infinitesimal perturbation in the deadtime, even though it may be nominally stable. Thus, robust stability and performance of the closed-loop have to be ensured for the application of the Smith system to be more effective. These issues are investigated in this section.

Robust stability

The Smith-predictor controller is referred to as being *robustly stable* if it is designed such that the closed-loop is stable for *all* members of a family of possible processes. Robust stability results are already available for IMC designs (Morari and Zafiriou, 1989). In this section, the robust stability of the Smith-predictor controller of Figure 1 is investigated by referring to these existing results. The following assumptions are made.

Assumption 1.

- The actual process $g_p(s)$ and the nominal process $g_{po}(s)$ do not have any unstable poles.
- The nominal closed-loop system $g_{yr}^*(s)$, between the set point and output, is stable.

Following is a useful lemma pertaining to IMC design (refer to Morari and Zafiriou (1989) for more details).

Lemma 1. Assume that the family of stable processes Π with norm-bounded uncertainty is described by

$$\Pi = \left\{ g_p : \left| \frac{g_p(j\omega)-g_{po}(j\omega)}{g_{po}(j\omega)} \right| = |l_m(j\omega)| \le \tilde{l}_m(\omega) \right\}, \quad (3)$$

where $\tilde{l}_m(\omega)$ is the bound on the multiplicative uncertainty $l_m(j\omega)$. Then the IMC system of Figure 2 (where q is stable) is robustly stable if and only if

$$|q(j\omega)g_{po}(j\omega)|\tilde{l}_m(\omega) < 1, \qquad \forall\omega. \quad (4)$$

The Smith-predictor controller can be rearranged into an equivalent IMC system (Figure 3), where $q = g_c/(1+g_c g_{ro})$.

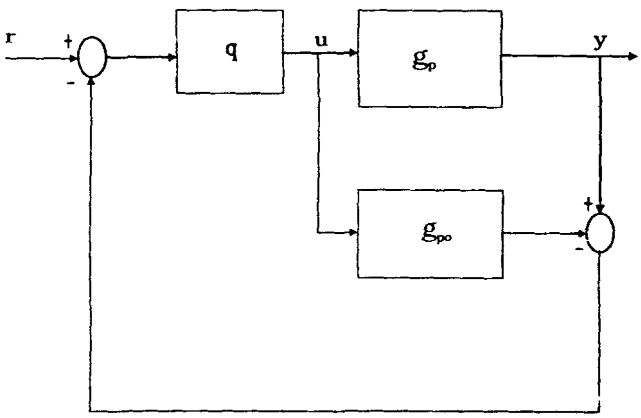

Figure 2. Internal model control.

Note that $g_{yr}^*(s) = q(s)g_{po}(s)$ is the nominal closed-loop transfer function between the setpoint and output, and under Assumption 1, q is stable. Theorem 1 is thus obtained, which provides a necessary and sufficient condition for the robust stability of the Smith-predictor controller.

Theorem 1. Assume that the family of stable processes Π is described by Eq. 3. Then, under Assumption 1, the Smith-predictor controller of Figure 1 is robustly stable, if and only if

$$|g_{yr}^*(j\omega)|\,\bar{l}_m(\omega) < 1, \qquad \forall\omega. \tag{5}$$

Remark 1. It is necessary to clarify what we mean when we say that Theorem 1 is not only sufficient for robust stability but also necessary. If condition Eq. 5 is violated, then in the set Π defined by Eq. 3, there exists a process g_p for which the closed-loop is unstable. If the set Π were obtained by approximating the true uncertainty regions with disks as described in Eq. 3, and therefore contains processes not present in the original uncertainty set, then Eq. 5 will be generally only sufficient for the original uncertainty set.

Remark 2. Equation 5 can be rewritten as

$$|g_{yr}^*(j\omega)| < \bar{l}_m^{-1}(\omega), \qquad \forall\omega.$$

Given the uncertainty bound $\bar{l}_m(\omega)$, the robust stability of the closed-loop Smith-predictor controller can be ensured by a proper design of the ideal closed-loop transfer function given by $g_{yr}^*(s)$ so that the magnitude-frequency response of $|g_{yr}^*(j\omega)|$ lies below that of $\bar{l}_m^{-1}(\omega)$, and Eq. 5 is satisfied.

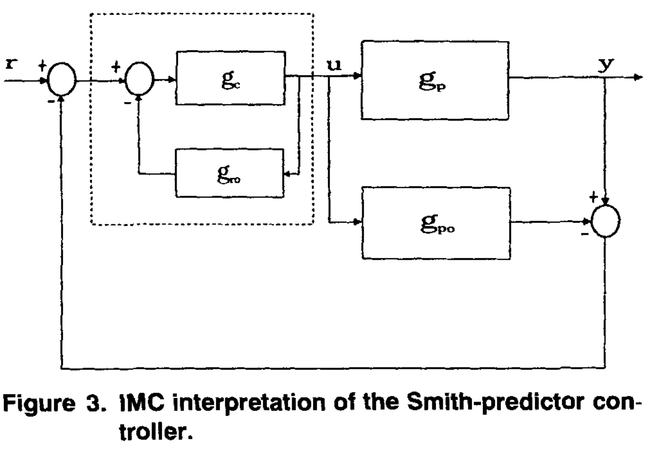

Figure 3. IMC interpretation of the Smith-predictor controller.

Practical stability

A weaker notion of robust stability is that of practical stability (Palmor, 1980). Practical stability deals with infinitesimal perturbations in the process dynamics, and it is important that any control system design has to be at least practically stable to be useful in a practical environment. In view of this, the condition for the practical stability of the closed-loop Smith-predictor controller of Figure 1 will now be derived.

Definition 1. The Smith-predictor controller of Figure 1 is said to be practically stable if and only if there exist positive numbers ω_m, δ such that the system is stable if

$$|l_m(j\omega)| = \left|\frac{g_p(j\omega)}{g_{po}(j\omega)} - 1\right| < \delta, \qquad 0 \le \omega < \omega_m. \tag{6}$$

Doyle and Stein (1981) point out that the uncertainty of process dynamics increases monotonically with frequency. In view of this, for the system to remain stable, mismatch is required here to be small in a finite frequency range. If $\omega_m > 0$ is chosen to be sufficiently large, and $\delta > 0$ sufficiently small, then $\bar{l}_m(\omega)$ can be made arbitrarily small for $0 \le \omega < \omega_m$ so that

$$|g_{yr}^*(j\omega)|\,\bar{l}_m(\omega) < 1, \qquad 0 \le \omega < \omega_m.$$

Such an $\omega_m \in (0,\infty)$ exists, and Theorem 1 is satisfied (i.e., $|g_{yr}^*(j\omega)|\,\bar{l}_m(\omega) < 1$, $\forall\omega$), if and only if

$$\lim_{\omega\to\infty} |g_{yr}^*(j\omega)|\,\bar{l}_m(\omega) < 1.$$

Thus, Corollary 1 follows.

Corollary 1. Under Assumption 1, the Smith-predictor controller of Figure 1 is practically stable if and only if

$$\lim_{\omega\to\infty} |g_{yr}^*(j\omega)|\bar{l}_m(\omega) < 1. \tag{7}$$

The relative degree (or pole excess) of a rational function is defined as the degree of the denominator polynomial minus the degree of the numerator polynomial, and it is always nonnegative if the function is proper. All practical processes involve some inherent dynamical lags so there is no direct feedthrough between the process input and output. Furthermore, delay systems may be described by

$$g_p(s) = g_r(s)e^{-sL},$$

where $g_r(s)$ is a delay-free rational function. Therefore, the following assumption is natural and reasonable.

Assumption 2. $g_r(s)$ is strictly proper, that is, $\deg(g_r) \ge 1$.

Under Assumptions 1 and 2, if $g_{ro}(s)$ is strictly proper, and $g_c(s)$ is proper, then as $\omega \to \infty$,

$$
\begin{aligned}
|g_{yr}^*(j\omega)l_m(j\omega)| &= \left|\frac{g_c(j\omega)}{1+g_c(j\omega)g_{ro}(j\omega)}[g_{po}(j\omega)-g_p(j\omega)]\right| \\
&\le \left|\frac{g_c(j\omega)}{1+g_c(j\omega)g_{ro}(j\omega)}\right| \cdot [\,|g_{ro}(j\omega)|\cdot|e^{-jL_o\omega}| \\
&\qquad + |g_r(j\omega)|\cdot|e^{-jL\omega}|\,] \to 0 < 1.
\end{aligned}
$$

From Corollary 1, the Smith-predictor controller thus designed is practically stable.

Corollary 2. Under Assumptions 1 and 2, the Smith-predictor controller of Figure 1 is practically stable if $g_{ro}(s)$ is strictly proper and $g_c(s)$ proper, that is,

$$\deg(g_{ro}) \ge 1, \qquad \deg(g_c) \ge 0.$$

Remark 3. In adaptive control systems, one of the usual requirements for robust adaptive control is prior knowledge of an upper bound on the order of the process. To ensure stability of the adaptive system, the process model used in the adaptive controller should have an order greater than or equal to this upper bound. In actual applications, this requirement is rarely satisfied, as practical systems are essentially of the distributed parameter type, and there are no finite bounds on the order of the process. In contrast, for the Smith-predictor controller, such a restrictive condition is not required, and practical stability is achieved with no constraint on the order of the process to be controlled. Corollary 2 shows that the practical stability of the Smith-predictor controller can be ensured by a proper selection of controller and model structure.

Remark 4. Practical stability, by definition, deals with small perturbations in process dynamics. With the establishment of Corollaries 1 and 2, it has been shown that the Smith-predictor controller can be designed by a proper structural selection to have stability robustness to the same extent as conventional PID controllers, a feature that is desirable in practical applications. Note that this implies that the Smith-predictor controller thus designed maintains stability for small deviations between the actual process dynamics and its nominal model during real-time operations. While a robust design method generally involves shaping the closed-loop transfer function to satisfy conditions like Eq. 5, Corollaries 1 and 2 are useful as an initial step in selecting the structure of the model and controller for practical stability.

Robust performance

Robust stability is desirable in a practical environment where model uncertainty is an important issue. However, robust stability alone is not enough. Even if Eq. 5 is satisfied for the family Π, there will exist a "worst case" process in Π for which the closed-loop system is on the verge of instability, and for which the performance is arbitrarily poor. Thus, it is necessary to ensure that some performance specifications are met for *all* processes in the family Π. Performance specifications stated in the H_∞ framework (Morari and Zafiriou, 1989) requires

$$\max_{g_p \in \Pi} \|SW\|_\infty = \max_{g_p \in \Pi} \sup_\omega |S(j\omega)W(j\omega)| < 1, \qquad (8)$$

where S is the sensitivity function defined as

$$S(s) = 1 - g_{yr}(s),$$

and W is the performance weight. In general, W^{-1} provides a bound on the sensitivity function S. For the simple choice of $W^{-1} = MP$ (the specified maximum peak of S), robust performance only requires the maximum peak of the sensitivity function to be less than MP with no bandwidth constraint. The reader may refer to Laughlin et al. (1987) for more details on the choice of the performance weight, W. As in the section on robust stability, an existing result in IMC design (Morari and Zafiriou, 1989) will be applied to induce results for the Smith-predictor controller.

Lemma 2. Assume that the family of stable processes Π with norm-bounded uncertainty is described by Eq. 3. Then the IMC system of Figure 2 (where q is stable) will meet the performance specification, Eq. 8, if and only if

$$|q(j\omega)g_{po}(j\omega)|\,\tilde{l}_m(\omega) + |[1-q(j\omega)g_{po}(j\omega)]W(j\omega)| < 1, \quad \forall\omega.$$

Using Lemma 2 and the IMC interpretation of the Smith-predictor controller, the following theorem is obtained that provides a necessary and sufficient condition for robust performance of the Smith-predictor controller.

Theorem 2. Assume that the family of stable processes Π is described by Eq. 3. Then under Assumption 1, the Smith-predictor controller of Figure 1 will meet the performance specification Eq. 8 if and only if

$$|g_{yr}^*(j\omega)|\,\tilde{l}_m(\omega) + |[1-g_{yr}^*(j\omega)]W(j\omega)| < 1, \qquad \forall\omega. \quad (9)$$

Remark 5. Equation 9 is necessary and sufficient for the norm-bounded uncertainty described in Eq. 3. In general, however, it is only sufficient when the true uncertainty region is only a portion of the uncertainty region described in Eq. 3.

Remark 6. Equation 9 can be rewritten as

$$\frac{|g_{yr}^*(j\omega)|}{1-|[1-g_{yr}^*(j\omega)]W(j\omega)|} < \tilde{l}_m^{-1}(\omega).$$

Given the uncertainty bound $\tilde{l}_m(\omega)$ and the performance weight $W(j\omega)$, the robust performance design is thus to design a proper ideal closed-loop transfer function $g_{yr}^*(s)$ so that the magnitude-frequency response of

$$\frac{|g_{yr}^*(j\omega)|}{1-|(1-g_{yr}^*(j\omega))W(j\omega)|}$$

lies below that of $\tilde{l}_m^{-1}(\omega)$ to satisfy Eq. 9. Depending on $\tilde{l}_m(\omega)$ and the specification $W(j\omega)$, if there is no $g_{yr}^*(j\omega)$ that satisfy Eq. 9, then $W(j\omega)$ may be too tight for the uncertainty present, and it may have to be relaxed.

Application: Robust Smith-Predictor Controller Design for a First-Order Deadtime System

In this section, an application of the robust Smith design method to first-order deadtime systems is developed to illustrate the robustness results obtained. This particular application is chosen because of its relevance in process control (Murrill, 1988; Seborg et al., 1989). Assume that a model described by

$$g_{po}(s) = \frac{k_o}{\tau_o s + 1} e^{-sL_o},$$

is available for the process $g_p(s) = [k/(\tau s + 1)]e^{-sL}$. Furthermore, assume the parameters in the model are uncertain,

$$k_l \le k \le k_u, \qquad \tau_l \le \tau \le \tau_u, \qquad L_l \le L \le L_u,$$

with

$$k_o = \frac{k_l + k_u}{2}, \qquad \tau_o = \frac{\tau_l + \tau_u}{2}, \qquad L_o = \frac{L_l + L_u}{2}.$$

Uncertainty bound, $\tilde{l}_m(\omega)$

Given the simultaneous uncertainties in gain k, time constant, τ, and deadtime L, an exact analytical expression for the bound $\tilde{l}_m(\omega)$ has been derived (Laughlin et al., 1987), and it is given in Eqs. 10–11:

$$\tilde{l}_m(\omega) = \left| \left(\frac{|k_o| + \Delta k}{|k_o|} \right) \left(\frac{j\tau_o \omega + 1}{j(\tau_o - \Delta\tau)\omega + 1} \right) e^{(j\Delta L \omega)} - 1 \right|, \qquad \forall \omega < \omega^*, \quad (10)$$

$$= \left| \left(\frac{|k_o| + \Delta k}{|k_o|} \right) \left(\frac{j\tau_o \omega + 1}{j(\tau_o - \Delta\tau)\omega + 1} \right) \right| + 1, \qquad \forall \omega \ge \omega^*, \quad (11)$$

where ω^* is defined implicitly by

$$\Delta L \omega^* + \arctan\left[\frac{\Delta\tau\omega^*}{1 + \tau_0(\tau_o - \Delta\tau)\omega^{*2}} \right] = \pi, \qquad \frac{\pi}{2} \le \Delta L \omega^* \le \pi, \quad (12)$$

and

$$\Delta k = |k_u - k_o| < |k_o|, \qquad \Delta\tau = |\tau_u - \tau_o| < |\tau_o|, \qquad \Delta L = |L_u - L_o| < |L_o|.$$

When only the deadtime L_o is uncertain, that is, $\Delta\tau = \Delta k = 0$, then $\omega^* = \pi/\Delta L$, and the preceding general expression simplifies to

$$\tilde{l}_m(\omega) = |e^{(j\Delta L \omega)} - 1|, \qquad \forall \omega < \frac{\pi}{\Delta L},$$

$$= 2, \qquad \forall \omega \ge \frac{\pi}{\Delta L}.$$

When only the gain k_o is uncertain, that is, $\Delta\tau = \Delta L = 0$, the expression simplifies to

$$\tilde{l}_m(\omega) = \frac{\Delta k}{|k_o|}, \qquad \forall \omega.$$

Controller design

A systematic two-step approach is adopted in the controller design. First, the model and controller structure are chosen to satisfy the practical stability condition given in Corollary 2. In this case, the first-order model is specified. Consider $g_c(s)$ chosen as a PI controller, that is,

$$g_c(s) = k_c\left(1 + \frac{1}{s\tau_i}\right), \quad (13)$$

with $\tau_i = \tau_o$; then it can be shown that

$$g_{yr}^*(s) = \frac{1}{\tau_{cl} s + 1} e^{-sL_o},$$

where $\tau_{cl} = \tau_i/(k_o k_c)$. Transfer function $g_{yr}^*(s)$ is stable. Furthermore, $\deg(g_c) = 0$, $\deg(g_{ro}) = 1$ so that following Corollary 2, the Smith-predictor controller thus designed is practically stable for any choice of k_c.

In the next step, k_c is to be chosen so that Eq. 5 or Eq. 9 is satisfied. Using the uncertainty bound $l_m(\omega)$ computed in the preceding subsection, this step basically entails the assignment of an optimal closed-loop bandwidth to the closed-loop transfer function $g_{yr}^*(s)$ based on the performance specification W.

Example. Assume that a nominal model is described by

$$g_{po}(s) = \frac{0.8}{s + 1} e^{-5s},$$

and all three parameters of the model are uncertain,

$$0.7 \le k \le 0.9, \qquad 0.8 \le \tau \le 1.2, \qquad 4 \le L \le 6.$$

It is desired to design the Smith-predictor controller with $g_c(s)$ as a PI controller for robust performance with the performance weight at $W^{-1} = 2$. With the controller parameters chosen as $(k_c, \tau_i) = (0.28, 1)$, Eq. 9 is satisfied since the magnitude-frequency response of

$$\frac{|g_{yr}^*(j\omega)|}{1 - |(1 - g_{yr}^*(j\omega))W(j\omega)|}$$

lies below that of $\tilde{l}_m^{-1}(j\omega)$ as shown in Figure 4.

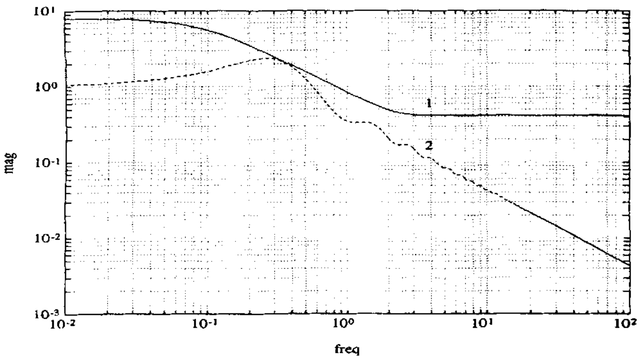

Figure 4. Magnitude-frequency response of (1) $\tilde{l}_m^{-1}(\omega)$, and (2) $\dfrac{|g_{yr}^*(j\omega)|}{1-|(1-g_{yr}^*(j\omega))W(j\omega)|}$.

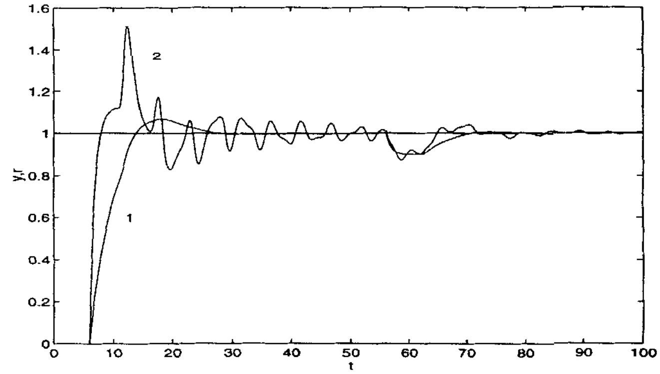

Figure 5. Performance of (1) the proposed robust Smith system, and (2) a nonrobust Smith system.

The performance of the Smith-predictor controller to a unit step change of the reference signal at $t=0$, and a 10% load disturbance at $t=50$, is shown in Figure 5 for the extreme case perturbation in the process parameters of $k=0.9$, $\tau=0.8$, and $L=6$. Clearly, the proposed system demonstrates a robust performance with tight control of the process variable toward asymptotic setpoint tracking and disturbance rejection. The performance of a Smith system designed without robustness is taken into consideration by Lee et al. (1994) and is provided for comparison.

Case Study: Robust Level Control of a Coupled-Tanks Apparatus

The robust design method for the Smith-predictor controller described in the previous section is applied to a real-time experiment on robust level control in a coupled-tanks system with transport delay, and some results are briefly described here. The experimental setup of the coupled-tanks system is shown in Figure 6, and a photograph of the apparatus is also shown in Figure 7.

The pilot scale process consists of two square tanks, Tank 1 and Tank 2, coupled to each other through an orifice at the bottom of the tank wall. The inflow (control input) is supplied by a variable-speed pump that pumps water from a reservoir into Tank 1 through a long tube. The orifice between Tank 1 and Tank 2 allows the water to flow into Tank 2. In the experiments, it is desirable to control the process with the voltage to drive the pump as input, and the water level in Tank 2 as process output. In the setup here, the effect of transport delay has been intentionally incorporated in the experimental setup by means of additional digital electronic hardware that cascades a time-delay into the open-loop process. An additional deadtime of 8.3 (time scale is in minutes) is simulated. This coupled-tanks pilot process has process dynamics that are representative of many fluid level control problems faced in the process control industry.

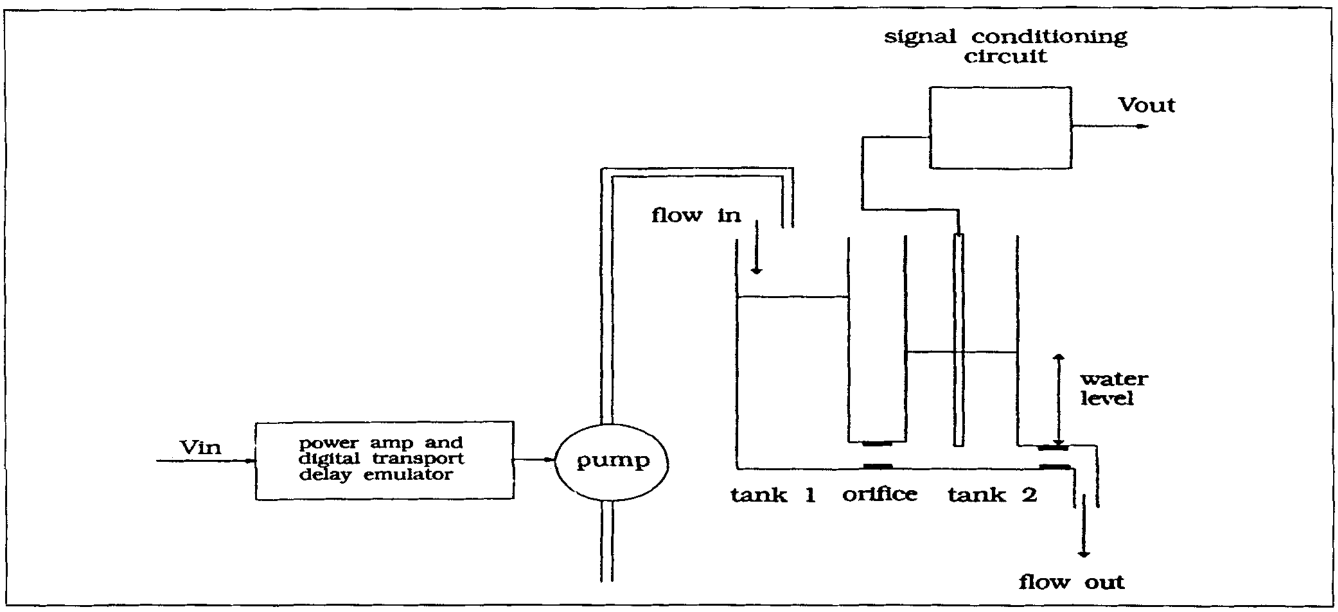

Figure 6. Coupled-tanks system.

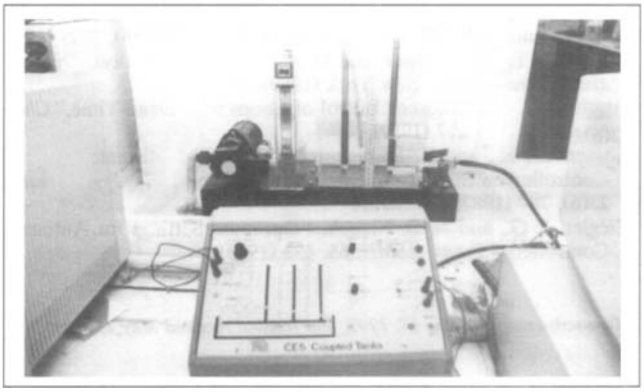

Figure 7. Coupled-tanks system.

Figure 8 shows that from $t = 0$ to $t = 232$, an autotuning procedure (Lee et al., 1994) is conducted on the process, after which the process is modeled as

$$g_{po}(s) = \frac{1.09}{1.55s + 1} e^{-8.5s}.$$

A 20% drift in the deadtime of the process is allowed, and the performance weight is specified as $W^{-1} = 3$. With $k_c = 1$, the robust performance condition is satisfied and verified in Figure 9. The Smith-predictor controller is thus commissioned with $(k_c, \tau_i) = (1, 1.55)$. A setpoint step change is introduced at $t = 232$, and at the same time, the process is perturbed by deliberately reducing the delay simulation by 20% from 8.3 to 6.9. Besides, a load disturbance is introduced at $t = 276$. The autotuning and the subsequent robust performance is shown in Figure 8.

Conclusions

Necessary and sufficient conditions for the practical stability, robust stability, and the robust performance are obtained for Smith-predictor controller with a norm-bounded uncertainty associated with the process model. Based on these conditions, a systematic two-step controller design approach is developed where, first, the model and controller structure is selected for practical stability, and, second, the optimal parameters of the controller are chosen to satisfy the performance specifications under the perturbed conditions.

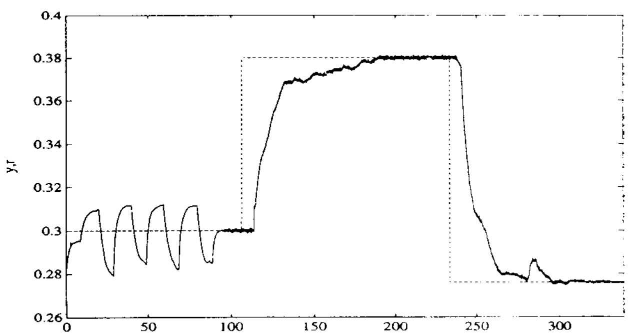

Figure 8. Autotuning and robust closed-loop performance—case study (time scale: min).

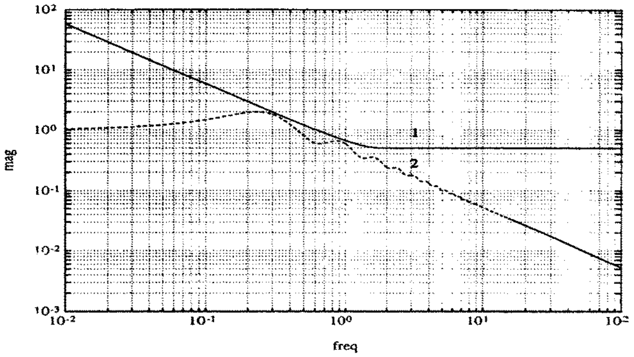

Figure 9. Magnitude-frequency response of (1) $\tilde{l}_m^{-1}(\omega)$, and (2) $\dfrac{|g_{yr}^*(j\omega)|}{1 - |(1 - g_{yr}^*(j\omega))W(j\omega)|}$—case study.

Notation

g_c = transfer function of the controller
g_p = transfer function of the process
g_{po} = transfer function of the process model
k_l = lower bound of k
k_o = static gain of the process model
k_u = upper bound of k
Δk = magnitude of $k_u - k_l$
L_l = lower bound of L
L_u = upper bound of L
ΔL = magnitude of $L_u - L_l$
q = transfer function of IMC controller
r = setpoint
s = Laplace transform variable
t = time variable
y = process output

Greek letters

δ = real number denoting amount of uncertainty
ω, ω^* = frequency variable
ω_m = real number denoting a frequency range
τ_l = lower bound of τ
τ_o = time constant of a first-order mode
τ_u = upper bound of τ
$\Delta\tau$ = magnitude of $\tau_u - \tau_l$

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